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FORM FACTORS OF SEMILEPTONIC M_{l3} DECAYS*

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ABSTRACT

We study in detail the kinematics of semileptonic M_{l3} ($M \rightarrow M' l \nu$) decay, considering only the new models preserving all the gauge symmetries of the Standard Model (SM), and assuming that neutrinos are massless and left-handed as in the SM. And we present a brief review of the recent theoretical progress on model independent constraints on the form factors of M_{l3} decays using dispersion relations. Finally we review a new parton model approach for semileptonic $B \rightarrow D(D^*) l \nu$ decays, by extending the inclusive parton model and by combining with the results of the heavy quark effective theory. We also obtain the slope of the Isgur-Wise function.

1. Introduction

In exclusive weak decay processes of hadrons, the effects of strong interaction are encoded in hadronic form factors. These decay form factors are Lorentz invariant functions which depend on the momentum transfer q^2 , and their behaviors with varying q^2 are dominated by non-perturbative effects of QCD.

Over the past few years, a great progress has been achieved in our understanding of the exclusive semileptonic decays of heavy flavors to heavy flavors¹. In the limit where the mass of the heavy quark is taken to infinity, its strong interactions become independent of its mass and spin, and depend only on its velocity. This provides a new $SU(2N_f)$ spin-flavor symmetry, which is not manifest in the theory of QCD. However, this new symmetry has been made explicit in a framework of the heavy

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quark effective theory (HQET)². In practice, the HQET and this new symmetry relate all the hadronic matrix elements of $B \rightarrow D$ and $B \rightarrow D^*$ semileptonic decays, and all the form factors can be reduced to a single universal function, the so-called Isgur-Wise (IW) function^{2,3}, which represents the common non-perturbative dynamics of weak decays of heavy mesons. However, the HQET cannot predict the values of the IW function over the whole q^2 range, though the normalization of the IW function is precisely known in the zero recoil limit. Hence the extrapolation of q^2 dependences of the IW function and of all form factors is still model dependent and the source of uncertainties in any theoretical model. Therefore, it is strongly recommended to determine hadronic form factors of M_{l3} ($M \rightarrow M' l \nu$) decay more reliably, when we think of their importance in theoretical and experimental analyses.

Although the Standard Model (SM) has been an enormous success in explaining all the known experimental data, it is generally anticipated that there is new physics in higher energy regions. One of the long-shot efforts to exploring such new physics could be searches for CP violation (or T violation) outside of the Cabibbo-Kobayashi-Maskawa (CKM) paradigm⁴. It is well known⁵ that measuring a component of muon polarization normal to the decay plane in $K_{\mu 3}$ ($K^+ \rightarrow \pi^0 \mu^+ \nu$) decay would signal T violation. It is also expected that the CKM phase does not induce the perpendicular muon polarization in $K_{\mu 3}$ decay. Therefore, measurements⁶ of these polarizations could be clear signatures of physics beyond the SM.

In Section 2, we study in detail the kinematics of semileptonic M_{l3} decay, considering only the new models preserving all the gauge symmetries of the SM, and assuming that neutrinos are massless and left-handed as in the SM. And in Section 3, we present a brief review of the recent theoretical progress on model independent constraints on the form factors of M_{l3} decays using *dispersion relations*. Finally in Section 4, we review a new parton model approach for semileptonic $B \rightarrow D(D^*) l \nu$ decays, by extending the inclusive parton model and by combining with the results of the HQET. We also obtain the slope of IW function.

2. Kinematics of semileptonic M_{l3} decay

The semileptonic hadron decays are very important in (i) determining the CKM matrix within the SM, and (ii) investigating new effects from physics beyond the SM. In this report, considering only the new models preserving all the gauge symmetries of the SM, and assuming that neutrinos are massless and left-handed as in the SM, we concentrate on the semileptonic decay of a pseudo-scalar meson M to a pseudo-scalar meson M' in the M rest frame

$$M(p) \rightarrow M'(p') + l(k) + \nu(k'), \quad (1)$$

where all the four-momenta are given in the parentheses. In the SM the semileptonic decay process (1) proceeds at quark level through a W boson exchange, while the

decay process may proceed through charged scalar/tensor exchanges in new physics beyond the SM as well.

The decay amplitude for the process $M \rightarrow M' l \nu$ can be parametrized in the factorized form as

$$\mathcal{M} \propto G_F V \left[H^\mu L_\mu \right], \quad (2)$$

where G_F is the Fermi decay constant and V is a CKM matrix element. While the leptonic current, L^μ , is completely determined, the determination of the hadronic current, H^μ , is limited due to the complicated strong interactions. However, translation and Lorentz invariances force H^μ to be of the form for a scalar or vector exchange:

$$\begin{aligned} H^\mu &= (p + p')^\mu f_+(t) + (p - p')^\mu f_-(t), \\ &= \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) f(t) + \frac{q^\mu}{q^2} d(t), \end{aligned} \quad (3)$$

where $q = p - p' = k + k'$ and $t = q^2$. In Eq. (3) we have introduced two different parametrizations, which are related as follows

$$f(t) = 2f_+(t), \quad d(t) = (m^2 - m'^2)f_+(t) + t f_-(t), \quad (4)$$

where $m(m')$ is the mass of the pseudo-scalar meson $M(M')$. Clearly, the so-called scalar form factor $d(t)$ vanishes in the limit where the hadronic current is conserved. For our convenience, the latter form-factor set $(f(t), d(t))$ will be employed in the followings.

It is now straightforward to calculate the absolute square of the decay amplitude (2), which can be cast into the form

$$|\mathcal{M}|^2 \propto G_F^2 |V_{12}|^2 \left[T_f |f|^2 + T_d |d|^2 + T_{fd} \mathcal{R}(f d^*) + A_{fd} \mathcal{I}(f d^*) \right]. \quad (5)$$

In general, the distributions, T_i ($i = f, d, fd$) and A_{fd} are functions of the four-momenta and the lepton polarization vector, which can be explicitly determined. First of all, we note that, when the summation over polarization of the lepton is taken, the distribution, A_{fd} , vanishes and the other distributions are given by

$$\begin{aligned} T_f &= \frac{1}{8t^2} \left\{ t(t - m_l^2) \left[(t + m^2 - m'^2)^2 - 4m^2 t \right] \right. \\ &\quad \left. - \left[(t + m_l^2)(t + m^2 - m'^2) - 4tx \right]^2 \right\}, \\ T_d &= \frac{m_l^2}{2t^2} (t - m_l^2), \\ T_{fd} &= \frac{1}{4t^2} \left[(t + m_l^2)(t + m^2 - m'^2) - 4tx \right], \end{aligned} \quad (6)$$

where $x = mE_l$ with the lepton energy E_l in the M rest frame. Secondly, we can see that three terms $|f|^2$, $|d|^2$, and $\mathcal{R}(fd^*)$, are independently separable due to the different dependence of the three distributions on the variable x . Of course, in this case, the lepton mass m_l should not be so small in order for the $|d|^2$ term to be measurable. On the other hand, the differential decay width is given in terms of t and x by

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{|\mathcal{M}|^2}{16m^3} dt dx, \quad (7)$$

where $m_l^2 \leq t \leq (m - m')^2$ and the scatter plot in t and x , which is called a Dalitz plot, has uniform phase space density.

With the lepton polarization into account, we find that the distribution A_{fd} in the M rest frame is nothing but the so-called triple vector correlation

$$A_{fd} = \frac{2m_l m}{t} \left[\vec{s}_l \cdot (\vec{k} \times \vec{p}') \right], \quad (8)$$

where \vec{s}_l is the lepton polarization vector.

T-invariance requires that the relative phase of f and d should be the same or, in other words, $\mathcal{I}(fd^*)$ should be zero. Conversely, any non-zero value of $\mathcal{I}(fd^*)$ would signal violation of T-violation⁶. It should be noted that *the T-violation effect can not be measured without the triple vector correlation* (8). Moreover, the T-violation effect is proportional to the mass of the lepton and requires the measurement of the lepton polarization. While the polarization of the electron is almost not measurable, the polarization of the muon or tau leptons can be easily determined via the angular or energy distributions of their decay products. In light of these features, it is clear that in the K meson case $K_{\mu 3}$ is by far more sensitive than $K_{e 3}$, and in the B meson case the τ lepton mode is most sensitive to $\mathcal{I}(fd^*)$.

3. Model-independent constraints on the form factors

The form factors at specific kinematic points can be approximately determined by the heavy quark symmetries for heavy meson decays and by the chiral symmetries for light meson decays. However, the event rate vanishes at the kinematic point so that the extrapolation of the form factor to other kinematical values QCD is required to determine the CKM matrix with good precision. Moreover, the form factor extrapolation is unavoidably accompanied by an uncertainty due to the choice of parametrization. Estimates of this uncertainty obtained by varying parametrizations suffer the same ambiguity as well. Recently, the possibility of obtaining model-independent bounds on the form factors $f(t)$ and $d(t)$ using dispersion relations has revived much interest^{7,8}. In this section we present a brief review of the model-independent constraints on meson form factors using dispersion relations.

The point^{9,10,11} is that the form factors $f(t)$ and $d(t)$ of the semileptonic decay process $M \rightarrow M' l \nu_l$ can be constrained by the knowledge of the two-point function

$$i \int d^4 x e^{iq \cdot x} \langle 0 | T(V^\mu(x) V^{\nu\dagger}(0)) | 0 \rangle = -(g^{\mu\nu} t - q^\mu q^\nu) \Pi_T(t) + g^{\mu\nu} \Pi_L(t), \quad (9)$$

in the deep Euclidean region, where $V^\mu(x)$ denotes the vector current responsible for the transition $M \rightarrow M'$. In order to render both sides of the relation (9) finite we employ the once-subtracted dispersion relations

$$\chi_{T,L}(Q^2) = \frac{\partial \Pi_{T,L}}{\partial q^2} \Big|_{q^2 = -Q^2} = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im} \Pi_{T,L}(t)}{(t + Q^2)^2}, \quad (10)$$

where the spectral functions $\text{Im} \Pi_{T,L}(t)$ are defined by the relation

$$\begin{aligned} & -(g^{\mu\nu} t - q^\mu q^\nu) \text{Im} \Pi_T(t) + g^{\mu\nu} \text{Im} \Pi_L(t) \\ &= \frac{1}{2} \int \sum_\Gamma d\rho_\Gamma (2\pi)^4 \delta(q - p_\Gamma) \langle 0 | V^\mu(0) | \Gamma \rangle \langle \Gamma | V^{\nu\dagger}(0) | 0 \rangle, \end{aligned} \quad (11)$$

with the summation over all possible hadron states Γ of equal flavour quantum numbers and with an integral over the phase space of allowed intermediate states. The hadron states Γ might be resonances or open MM' states in a specific isospin channel determined by the quantum numbers of the relevant vector current V^μ . In some cases the open MM' state is precisely the lowest hadronic state contributing to the absorptive amplitude. Up to an overall Clebsh-Gordon coefficient η , the function $\langle 0 | V^\mu(0) | MM' \rangle$ is *the same analytic function* as $\langle M' | V^\mu(0) | M \rangle$ which appears in the decay $M \rightarrow M' l \nu_l$, i.e.,

$$\langle 0 | V^\mu(0) | M \bar{M}' \rangle = \eta \left\{ \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) f(t) + \frac{q^\mu}{q^2} d(t) \right\}, \quad (12)$$

where the regime of the variable t is $(m + m')^2 \leq t \leq \infty$.

A judicious choice of the indices μ and ν makes the spectral functions a sum of positive definite terms, so one can obtain strict inequalities by concentrating on the term with intermediate states of MM' pairs. In particular, the intermediate open state $M \bar{M}'$ gives the contribution

$$\begin{aligned} \text{Im} \Pi_T(t) &\geq \frac{\eta^2}{16\pi t} \sqrt{(t - t_-)(t - t_+)} \left[\frac{|d(t)|^2}{t^2} + \left(1 - \frac{t_-}{t}\right) \left(1 - \frac{t_+}{t}\right) \frac{|f(t)|^2}{4} \right], \\ \text{Im} \Pi_L(t) &\geq \frac{\eta^2}{16\pi t^2} \sqrt{(t - t_-)(t - t_+)} |d(t)|^2, \end{aligned} \quad (13)$$

for $t \geq t_+$ with $t_+ = (m + m')^2$ and $t_- = (m - m')^2$. Clearly, the inequalities become tighter with more possible contributions to the right side, which are always positive. Eqs. (10) and (13) allow us to obtain the inequalities on $\chi_{T,L}(Q^2)$, e.g.,

$$\chi_L(Q^2) \geq \frac{\eta^2}{16\pi^2} \int_{t_+}^\infty dt \frac{\sqrt{(t - t_-)(t - t_+)}}{t^2(t + Q^2)^2} |d(t)|^2 \equiv J(Q^2). \quad (14)$$

The spectral function $\chi_{T,L}(Q^2)$ can be computed reliably from perturbative for Q^2 far from the resonance region and their lowest-order expressions from a quark one-loop diagram are

$$\begin{aligned}\chi_T(Q^2) &= \frac{3}{4\pi^2} \int_0^1 dx \frac{2x^2(1-x)^2}{x(1-x)Q^2 + m_q^2x + m_{q'}^2(1-x)}, \\ \chi_L(Q^2) &= \frac{3}{4\pi^2} (m_q - m_{q'}) \int_0^1 dx \frac{x(1-x)[m_qx - m_{q'}(1-x)]}{x(1-x)Q^2 + m_q^2x + m_{q'}^2(1-x)},\end{aligned}\quad (15)$$

where the quark q (q') is inside M (M').

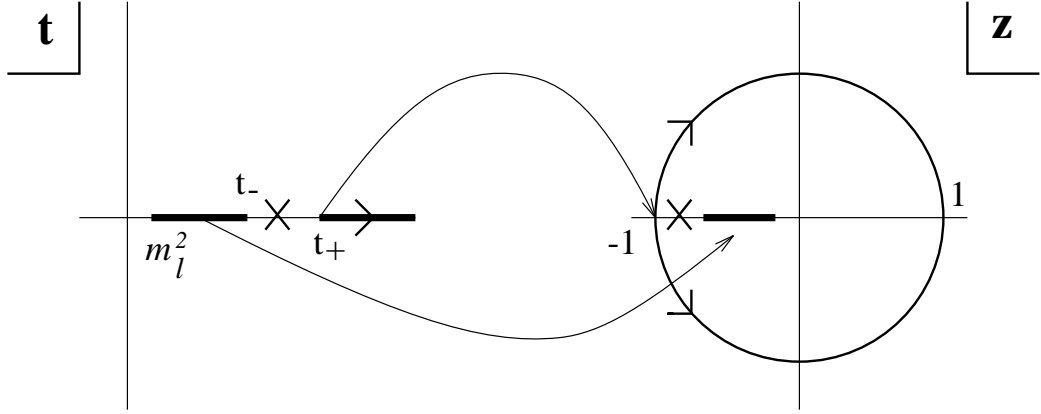


Fig. 1. Mapping of the cut t -plane into the unit disc by the conformal transformation.

Using knowledge of the analytic structure of the form factors plus the bounds (13) one can derive model-independent bounds on the form factors $f(t)$ and $d(t)$ in the physical region of semileptonic decay, $m_l^2 \leq t \leq (m - m')^2$. To this end let us map the complex t plane onto the unit disc $|z| \leq 1$ by $\sqrt{(t_+ - t)/(t_+ + Q^2)} = (1 + z)/(1 - z)$ (For the mapping, see Fig. 1), and then we can express $J(Q^2)$ as the norm squared of an analytic function,

$$J(Q^2) = ||h||^2 \equiv \frac{1}{2\pi} \int_0^{2\pi} d\theta |\varphi(e^{i\theta})d(e^{i\theta})|^2, \quad h(z) = \varphi(z)d(z), \quad (16)$$

on the unit disc of the complex z -plane with an explicitly determined function $\varphi(z)$.

With the inner product of functions defined on the unit disc as

$$\langle g_1(z) | g_2(z) \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta g_1^*(e^{i\theta}) g_2(e^{i\theta}), \quad (17)$$

we find that, for $g_1(z) = 1/(1 - \lambda^* z)$ and $g_2(z) = h(z)$, $\langle g_1(z)|h(z) \rangle = d(\lambda)\varphi(\lambda)$, d can be determined¹¹ with an appropriate λ at any point of the physical region $m_l^2 \leq t \leq t_-$ for the semileptonic decay $M \rightarrow M' l \nu_l$. Furthermore, the 2×2 positive semidefinite matrix of $A_{ij} = \langle g_i(z)|g_j(z) \rangle$, satisfying $\det(A_{ij}) \geq 0$, provides us with the model-independent bound on $d(\lambda)$

$$|d(\lambda)|^2 \leq \frac{J(Q^2)}{(1 - |\lambda|^2)|\varphi(\lambda)|^2} \leq \frac{\chi_L(Q^2)}{(1 - |\lambda|^2)|\varphi(\lambda)|^2}. \quad (18)$$

The bounds on the form factor $f(t)$ can be obtained through the same approach as well.

To summarize, the main aspects of the model-independent perturbative QCD method are as follows.

- The momentum transfer squared Q^2 should be far from the resonance region for reliable perturbative QCD calculations. For example, for the heavy B meson $Q^2 = 0$ can be taken⁷ because of large resonance mass compared to the QCD scale. However, for the light K or π mesons, the value of Q^2 should be large⁹.
- The spectral functions can be further saturated with more contributions included. However, the inclusion of resonances^{7,8} can cause uncertainties due to no precise information on their decay constants and masses. Of course, resonances with much larger than t_+ can be safely ignored. In this light, the $K \rightarrow \pi$ mode⁷ is a very good system because of no resonances for $t \leq t_+$. The $B \rightarrow \pi$ mode involves⁷ one resonance B^* , while the $B \rightarrow D$ mode involves a lot of resonances, which should be included for reliable predictions on the form factors.
- It is naturally expected that any theoretical or experimental inputs^{7,8} of the form factors $f(t)$ and $d(t)$ within the physical region should give much stronger constraints on the form factors.

As briefly reviewed so far, the perturbatively calculable two-point functions in the deep Euclidean region provide us with model-independent bounds on the form factors of semileptonic meson decays and the constraints can get much more stringent with more theoretical and experimental determinations of the form factors at specific kinematic points. A systematic investigation along this line is in progress¹².

4. Parton model approach for semileptonic $B \rightarrow D(D^*) l \nu$

In this part we developed the parton model approach for exclusive semileptonic B decays to D , D^* , and predicted the q^2 dependences of all form factors. For the details of this exclusive parton model approach, see Ref. Kim et al.¹³. Previously the parton model approach has been established to describe inclusive semileptonic B

decays¹⁴, and found to give excellent agreements with experiments for electron energy spectrum at all energies.

While many attempts^{15,16} describing exclusive B decays often take the pole-dominance ansätze as behaviors of form factors with varying q^2 , in our approach they are derived by the kinematical relations between initial b quark and final c quark. According to the Wirbel et al. model¹⁵, which is one of the most popular models to describe exclusive decays of B mesons, the hadronic form factors are related to the meson wavefunctions' overlap-integral in the infinite momentum frame, but in our model they are determined by integral of the fragmentation functions, which are at least experimentally measurable.

We now develop the parton model approach for exclusive semileptonic decays of B meson by extending the inclusive parton model, and by combining with the results of the HQET. Theoretical formulation of this approach is, in a sense, closely related to Drell-Yan process, while the parton model of inclusive B decays is motivated by deep inelastic scattering process. And the bound state effects of exclusive B decays are encoded into the hadronic distribution functions of partons inside an initial B meson and of partons of a final state resonance hadron. Then, the Lorentz invariant hadronic decay width can be obtained using the hadronic distribution functions $f_{D,B}(x)$,

$$E_B \cdot d\Gamma(B \rightarrow D(D^*)e\nu) = \int dx \int dy f_B(x) E_b \cdot d\Gamma(b \rightarrow ce\nu) f_D(y) . \quad (19)$$

The first integral represents the effects of motion of b quark within B meson and the second integral those of c quark within D meson. The variables x and y are fractions of momenta of partons to momenta of mesons,

$$p_b = xp_B , \quad p_c = yp_D , \quad (20)$$

in the infinite momentum frame. The functions $f_B(x)$ and $f_D(y)$ are the distribution function of b quark inside B meson, and the fragmentation function of c quark to D meson respectively. Since the momentum fractions and the distribution functions are all defined in the infinite momentum frame, we have to consider the collider Lorentz invariant quantity, $E \cdot d\Gamma$, to use at any other frame.

The distribution function can be identified with the fragmentation function for a fast moving b -quark to hadronize into a B meson in the infinite momentum frame. In general the distribution and fragmentation functions of a heavy quark ($Q = t, b, c$) in a heavy meson (Qq), which are closely related by a time reversal transformation, are of very similar functional forms, and peak both at large value of x . Brodsky et al.¹⁷ have calculated the distribution function of a heavy quark, which has the same form as Peterson's fragmentation function¹⁸. Therefore, here we follow the previous works¹⁴ to use the Peterson's fragmentation function for both distributions, $f_B(x)$ and $f_D(y)$. It has the functional form:

$$f_Q(z) = N_Q z^{-1} \left(1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z} \right)^{-2} , \quad (21)$$

where N_Q is a normalization constant, and Q denotes b or c quark. This functional form is not purely ad hoc., but motivated by general theoretical arguments, that the transition amplitude for a fast moving heavy quark Q to fragment into a heavy meson (Qq) is proportional to the inverse of the energy transfer ΔE^{-1} . When we use this function, we have the advantage that the parameter ϵ_Q of this function has been determined from high energy experiments. Thus we can replace uncertain wave functions of heavy mesons by the experimentally measurable fragmentation functions.

From Lorentz invariance we write the matrix element of the decay $\bar{B} \rightarrow De\bar{\nu}$ in the form

$$\langle D|J_\mu|B \rangle = f_+(q^2)(p_B + p_D)_\mu + f_-(q^2)(p_B - p_D)_\mu \quad , \quad (22)$$

and in terms of the HQET

$$\langle D(v')|J_\mu|B(v) \rangle = \sqrt{m_B m_D} (\xi_+(v \cdot v')(v + v')_\mu + \xi_-(v \cdot v')(v - v')_\mu) \quad . \quad (23)$$

Due to the conservation of leptonic currents, the form factors multiplied by q_μ do not contribute. Then the hadronic tensor is given by

$$\begin{aligned} H_{\mu\nu} &= \langle D|J_\mu|B \rangle \langle D|J_\nu|B \rangle^* \\ &= 2 |f_+(q^2)|^2 (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \quad , \end{aligned} \quad (24)$$

and can be expressed by the IW function,

$$H_{\mu\nu} = R^{-1} |\xi(v \cdot v')|^2 (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \left(1 + \mathcal{O}\left(\frac{1}{m_Q}\right) \right) \quad , \quad (25)$$

where

$$R = \frac{2\sqrt{m_B m_D}}{m_B + m_D} \quad .$$

By comparing the matrix element of the decay $\bar{B} \rightarrow De\bar{\nu}$ in the parton model approach and the matrix element in terms of the HQET, we can define the function $\mathcal{F}(q^2)$ as

$$\mathcal{F}(q^2) \equiv \int dx f_B(x) f_D(y(x, q^2)) xy^3(x, q^2) \quad . \quad (26)$$

For given q^2 in our parton picture, the function $\mathcal{F}(q^2)$ measures the weighted transition amplitude, which is explicitly given by the overlap integral of distribution functions of initial and final state hadrons. Comparing the hadronic tensor of HQET and that of parton model approach, the IW function is calculated

$$|\xi(v \cdot v')|^2 \left(1 + \mathcal{O}\left(\frac{1}{m_Q}\right) \right) = 4N \cdot R \cdot \mathcal{F}(v \cdot v') \quad . \quad (27)$$

Finally the q^2 spectrum is given by

$$\frac{d\Gamma(\bar{B} \rightarrow De\bar{\nu})}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{96\pi^3 m_B^3} \mathcal{F}(q^2) \left((m_B^2 - m_D^2 + q^2)^2 - 4m_B^2 q^2 \right)^{3/2} . \quad (28)$$

We can also obtain the decay spectrum for $B \rightarrow D^* l \nu$

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D^* e \bar{\nu})}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^5} \mathcal{F}(q^2) \left((m_B^2 - m_{D^*}^2 + q^2)^2 - 4m_B^2 q^2 \right)^{1/2} \\ &\times \left[m_B^2 W_1(q^2) \left((m_B^2 - m_{D^*}^2 + q^2)^2 - m_B^2 q^2 \right) \right. \\ &\quad \left. + \frac{3}{2} m_B^2 W_2(q^2) (m_B^2 - m_{D^*}^2 + q^2) + 3m_B^2 W_3(q^2) \right] , \quad (29) \end{aligned}$$

where

$$\begin{aligned} W_1(q^2) &= -N_1 \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) , \\ W_2(q^2) &= N_1 (m_B^2 - m_{D^*}^2 + q^2) \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) - 2N_3 q^2 \left(1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) , \\ W_3(q^2) &= -N_1 m_B^2 q^2 \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) \\ &\quad + N_3 q^2 (m_B^2 - m_{D^*}^2 + q^2) \left(1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) \\ &\quad + N_2 q^2 (m_B^2 + m_{D^*}^2 - q^2) \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) , \quad (30) \end{aligned}$$

and $\mathcal{F}(q^2)$ is defined in (26).

The result is plotted in Fig. 2, also compared with the recent CLEO data¹⁹. The thick solid line is our model prediction with the parameters ($\epsilon_b = 0.004$, $\epsilon_c = 0.04$) for the heavy quark fragmentation functions¹⁸, the thin solid line the Wirbel et al. model prediction¹⁵, and the dotted line the Körner et al. model prediction¹⁶. We also obtain the values of the slope parameter $\hat{\rho}$ within the parton model framework,

$$\hat{\rho}^2 = 0.582 - 0.896 ,$$

which are compatible with the average value measured by experiments²⁰,

$$\hat{\rho}^2 = 0.87 \pm 0.12 .$$

For more details of predictions from this exclusive parton model approach, see Ref. Kim et al.¹³.

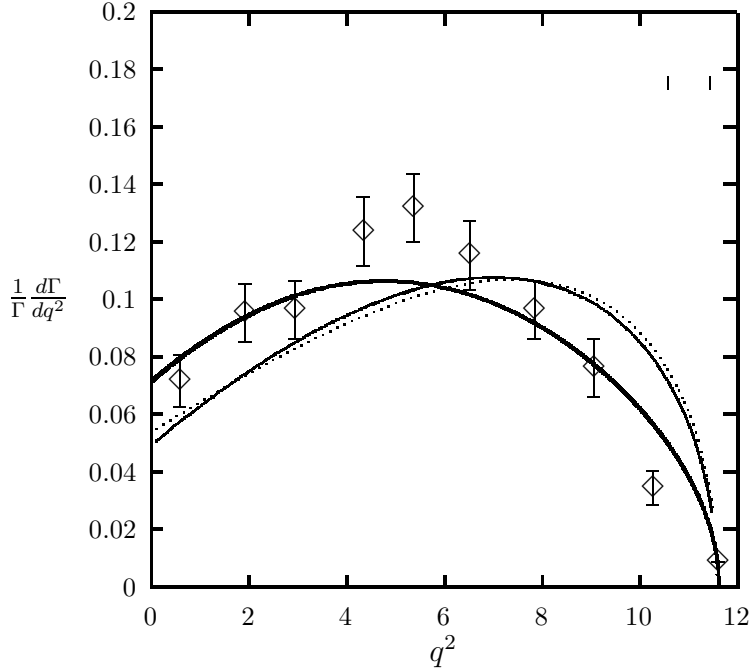


Fig. 2. q^2 spectrum in $B \rightarrow D^* e \nu$ decays.

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